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# Teaching Robot Kinematics For Engineering Technology Students Using a Created Three-Dimensional Robot and Camera

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## Teaching Robot Kinematics for Engineering Technology Students Using a Created Three-Dimensional Robot and a Camera

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# Teaching Robot Kinematics for Engineering Technology Students Using a Visualized Three-Dimensional Robot and a Camera

## **Abstract**

Teaching robot kinematics is important to engineering technology students in the robot automation. The study can help students not only in the coordinate transformation principles from a joint to its following joint in a robot, but also in relating the coordinate systems between a robot and a machine vision system. While students can utilize math software to compute robot kinematic transformations, they have problems verifying their answers. In this paper, a three dimensional vertically articulated robot is created to help students visualize the location and orientation of the end effector. Students can check their robot kinematic answers based on the joint encoder values set up at each joint. In addition, a camera is also mounted on the robot for the students to relate an object location from the camera coordinate system to the robot world frame.

## **1. Introduction**

A robot is typically connected by a series of links and joints. A robot hand, or end effector, is attached at the end of the wrist joint to pick up objects, transfer parts, and perform useful tasks<sup>1</sup>. Depending on the number of freely-movable joints available in a robot, it can move to multiple directions and can be called multiple degrees of freedom (DOF)<sup>2</sup>. Figure 1<sup>3</sup> shows a 2-DOF robot as it has two joint variables,  $\Theta_1$  and  $\Theta_2$ . Figure 2<sup>4</sup> shows a typical 6-DOF industrial robot with the first three joints (waist, shoulder, and elbow) controlling the motion to a target location, and the last three joints (roll, pitch, and yaw) controlling the orientation of the end effector, as shown in Figure 3<sup>5</sup>.

Robot kinematics is one of the main topics in teaching robot automation class. It studies the movement of the location and orientation of any link from the world frame in a robot manipulator<sup>6</sup>. As the robot performs works using its end effector, the location and orientation of the end effector based on the robot world frame is of particular interest when joint variables are given. The [4x4] homogeneous transformation matrices<sup>3,7</sup> are used in teaching this class mainly based on the following two reasons:

- a. It can well represent the location and orientation of a rigid body in a math form.
- b. Since it is a square  $[4 \times 4]$  matrix, it can be placed in any order of matrix multiplications.

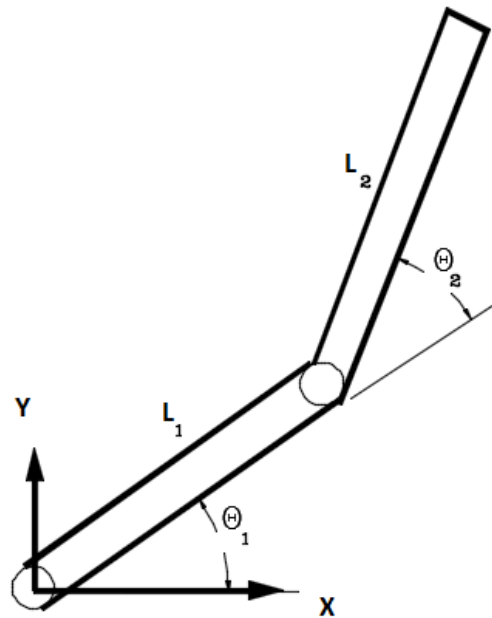


Figure 1: A 2-DOF Robot

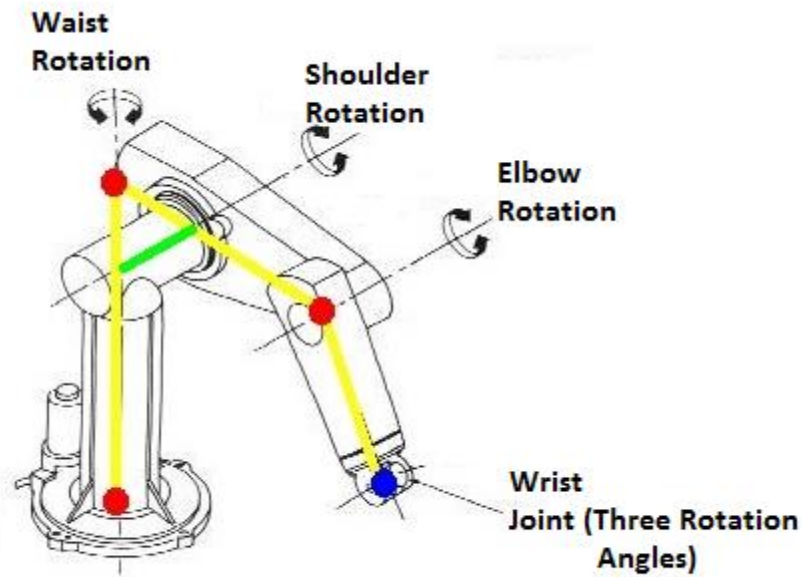


Figure 2: A Typical 6-DOF Industrial Robot

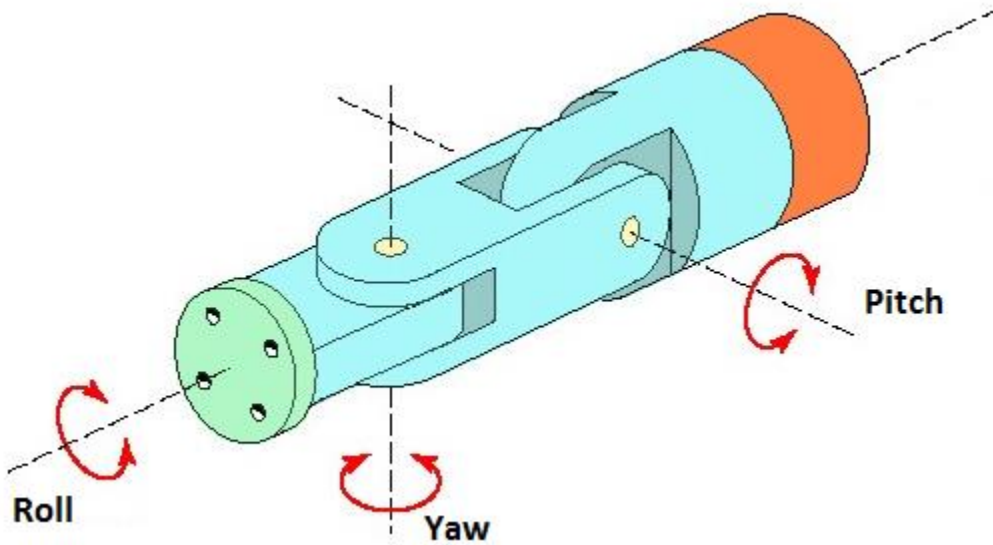


Figure 3: The Wrist Joints of a Robotic Hand

However, as the matrix multiplication in general is not commutative, students have problems calculating correct answers even though they are allowed to use math

software<sup>8</sup> for homework assignments. Thus, a visualized tool is presented for students to check their answer before submitting their assignments.

## 2. Homogeneous Transformation Matrices

A homogeneous transformation matrix, which can be used to define a local frame, is listed as following<sup>5</sup>:

$$\begin{bmatrix} n_x & o_x & a_x & d \\ n_y & o_y & a_y & e \\ n_z & o_z & a_z & f \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Figure 4 shows the local frame based on the world frame XYZ. Point  $(d,e,f)$ , based on the robot world frame, is the origin of this local frame. The following three unit vectors  $\mathbf{n}$ ,  $\mathbf{o}$ , and  $\mathbf{a}$ , can be expressed as:

$$\vec{n} = n_x \vec{i} + n_y \vec{j} + n_z \vec{k} \quad (1)$$

$$\vec{o} = o_x \vec{i} + o_y \vec{j} + o_z \vec{k} \quad (2)$$

$$\vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k} \quad (3)$$

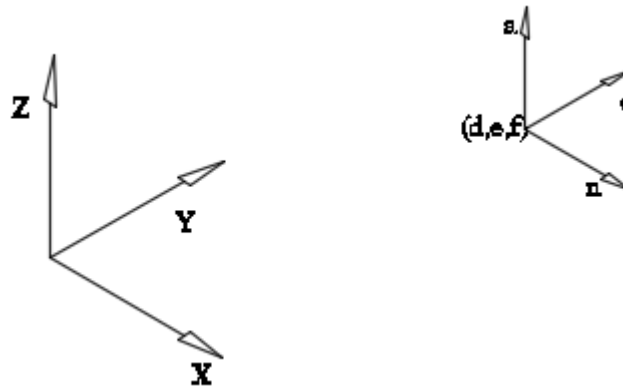


Figure 4: Local Frame Based on the World Frame

Where  $n_x$ ,  $n_y$ , and  $n_z$  represent a unit vector component for the local  $x$  vector along the robot world frame;  $o_x$ ,  $o_y$ , and  $o_z$  represent a unit vector component for the local  $y$  vector along the robot world frame;  $a_x$ ,  $a_y$ , and  $a_z$  represent a unit vector component for the local  $z$  vector along the robot world frame.

To completely describe the motion of each transformation, the following six elementary homogeneous transformation matrices are used in teaching robot kinematics<sup>6</sup>:

$$Trans(x, d) = \begin{bmatrix} 1 & 0 & 0 & d \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

$$Trans(y, e) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & e \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

$$Trans(z, f) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & f \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

$$Rot(x, \alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) & 0 \\ 0 & \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

$$Rot(y, \beta) = \begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

$$Rot(z, \gamma) = \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 & 0 \\ \sin(\gamma) & \sin(\gamma) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (9)$$

## 2.1. Example Using a Visual Tool to Check Elementary Transformation Matrices

To help students verify their answer using a CAD system, the following example is given:

**Problem:** Please determine the location of a point (1,2,3) after it rotates 60° around the Z axis of the world frame.

**Solution:** The following four steps are used to help students verify their solutions:

**Step 1:** List the equation using elementary transformation matrices, as shown in Equation (10).

$$\text{Rot}(Z, 60^\circ)\text{Trans}(x,1)\text{Trans}(y,2)\text{Trans}(z,3) \quad (10)$$

**Step 2:** Apply math software to obtain the solution, as shown in Figure 5.

```
a=60
RotZ1=[cosd(a) -sind(a) 0 0;sind(a) cosd(a) 0 0; 0 0 1 0; 0 0 0 1]
TransX1=[1 0 0 1; 0 1 0 0; 0 0 1 0; 0 0 0 1]
TransY2=[1 0 0 0; 0 1 0 2; 0 0 1 0; 0 0 0 1]
TransZ3=[1 0 0 0; 0 1 0 0; 0 0 1 3; 0 0 0 1]
A1= RotZ1*TransX1*TransY2*TransZ3

A1 =

    0.5000    -0.8660         0    -1.2321
    0.8660     0.5000         0     1.8660
         0         0     1.0000     3.0000
         0         0         0     1.0000
```

Figure 5: Using Math Software to Obtain the Answer

**Step 3:** Using a CAD system, create a point (1,2,3) based on the world frame, as shown in Figure 6.



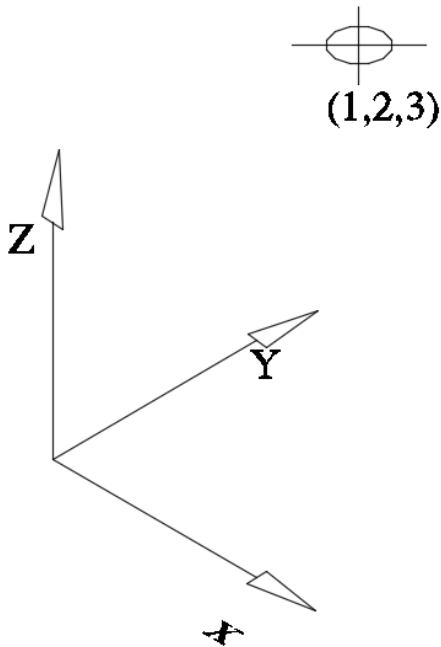


Figure 6: Point (1,2,3) Created in a CAD System

**Step 4:** Rotate Point (1,2,3) by  $60^\circ$  around Z axis in the CAD system and verify the location of the point, as shown in Figure 7.

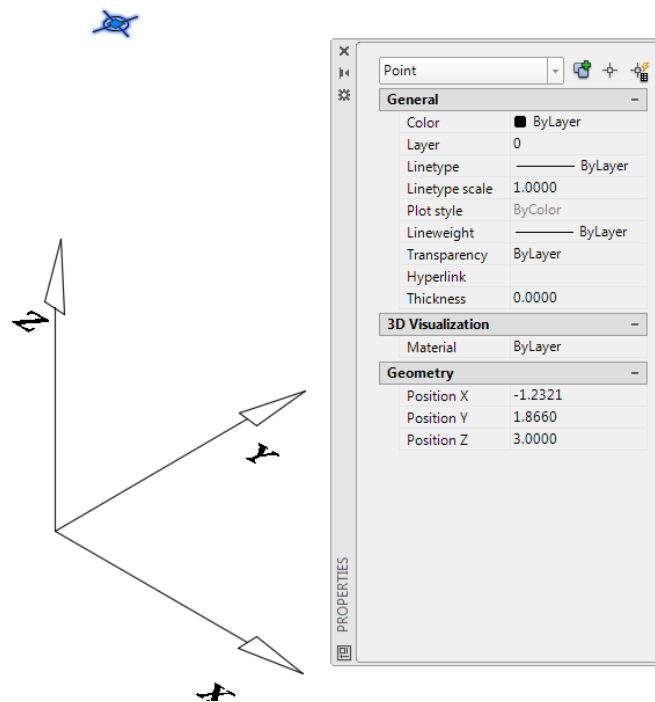


Figure 7: Verify Point Location after Point (1,2,3) Rotates  $60^\circ$  around the Z axis

**Step 5:** Compare the answers in Step 2 and Step 4. Both have the same answer: (-1.2321, 1.866, 3.00). If the answers are different and if the operation in Step 4 is confirmed correctly, go back to check Steps 1 and 2.

### 3. A Visualized Three-Dimensional Robot for Robot Kinematics

To help students verify their solutions for robot kinematics, Figure 8 shows a three-dimensional vertically articulated robot when all joint variables are at zero positions. Dimensions of each link are also provided. The origins of each link are shown in Figure 9. For students to verify their kinematic solutions, a verifying point (VP) is drawn at the origin of the end effector, as shown in Figure 10.

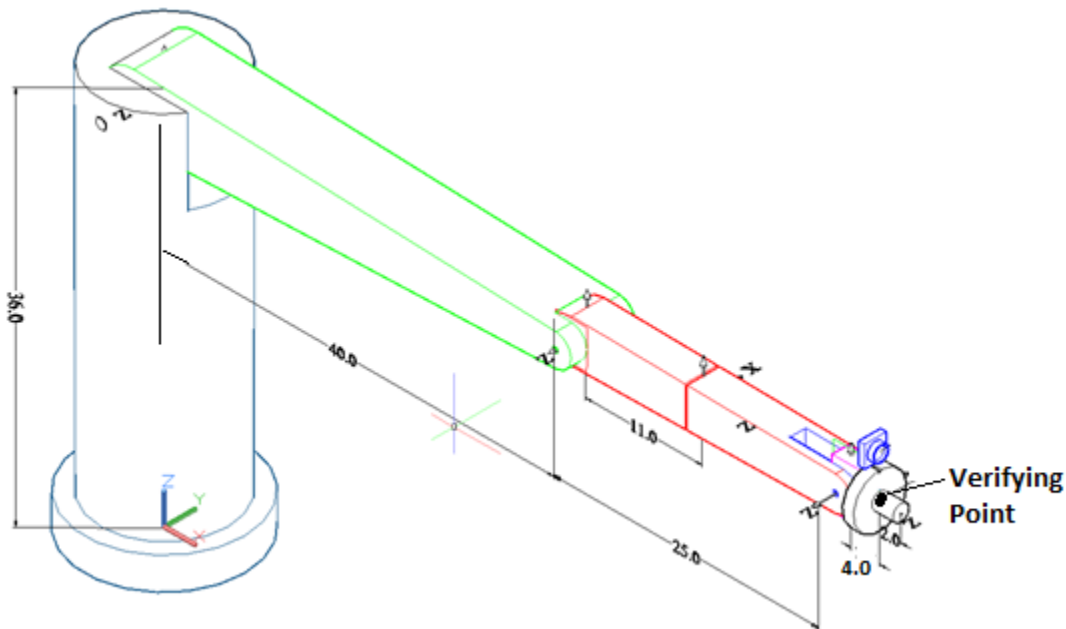


Figure 8: A Visualized Three-Dimensional Robot

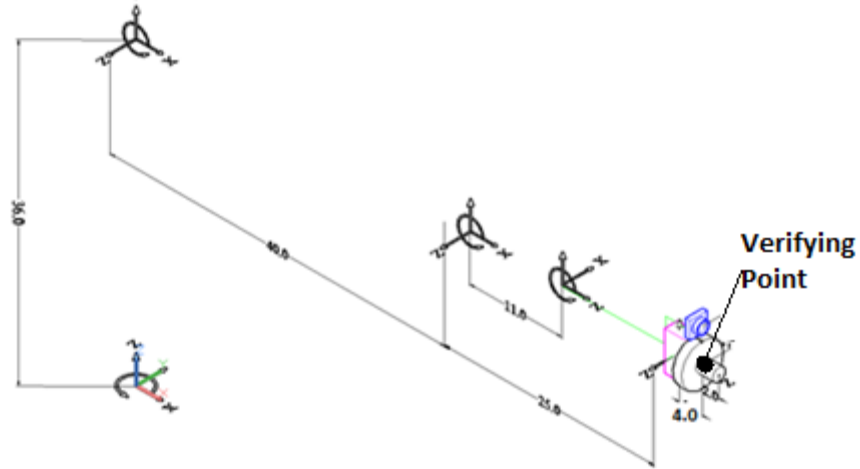


Figure 9: Joint Origins of Each Link

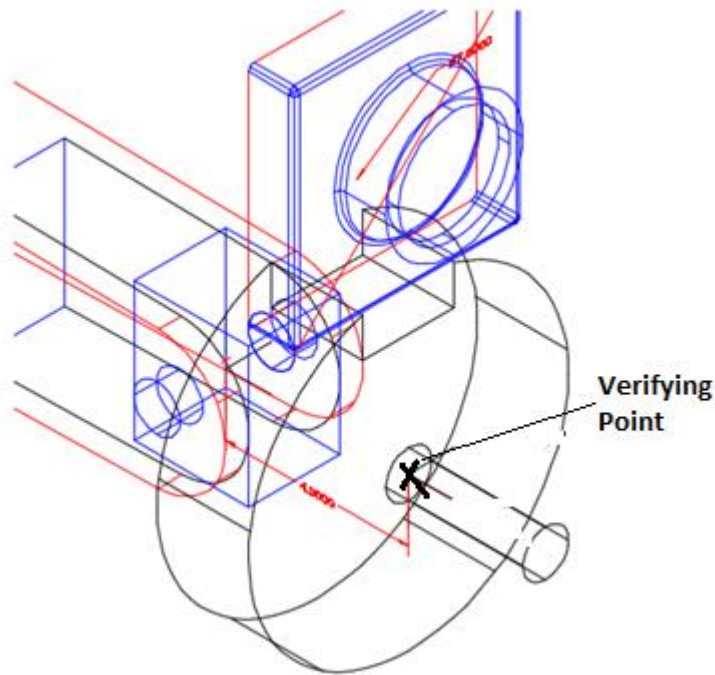


Figure 10: A Verifying Point Created to Verify the Location of the Wrist Joint

### 3.1. Robot Kinematics

To perform the transformation from one joint to its following joint, the Denavit Hartenberg (DH) transformation<sup>6</sup>, shown in Equation (11), is applied:

$$A_i = Rot(z, \theta_i) Trans(z, d_i) Trans(x, a_i) Rot(x, t_i) \quad (11)$$

Where  $A_i$  is the transformation from Joint  $i$  to Joint  $i+1$ ;  $\theta_i$  is the joint variable when the joint is a revolution joint;  $d_i$  and  $a_i$  can be the link lengths depending on the robot configuration;  $t_i$  represents the twist angle between Joint  $i$  and Joint  $i+1$ . In some particular cases,  $Rot(x, t_i)$  can be replaced with  $Rot(y, t_i)$  when the twist angle is along the Y axis.

From the information provided from Figure 9, the  $A_i$  matrix for each link can be expressed as following:

$$A_1 = Rot(z, \theta_1) Trans(z, 36) Rot(x, 90^\circ) \quad (12)$$

$$A_2 = Rot(z, \theta_2) Trans(x, 40) \quad (13)$$

$$A_3 = Rot(z, \theta_3) Trans(x, 11) Rot(Y, 90^\circ) \quad (14)$$

$$A_4 = Rot(z, \theta_4) Trans(z, 14) Rot(y, -90^\circ) \quad (15)$$

$$A_5 = Rot(z, \theta_5) Rot(Y, 90^\circ) \quad (16)$$

$$A_6 = Rot(z, \theta_6) Trans(z, 4) \quad (17)$$

The following equation is applied to calculate the Verifying Point (VP) based on the robot world frame:

$$T_{VP} = A_1 A_2 A_3 A_4 A_5 A_6 \quad (18)$$

The following joint variables are used to compute the location of the VP:  $\theta_1 = 30^\circ$ ;  $\theta_2 = 45^\circ$ ;  $\theta_3 = -60^\circ$ ;  $\theta_4 = -20^\circ$ ;  $\theta_5 = -60^\circ$ . Figure 11 shows the execution of the math script for the VP: (46.94, 25.73, 54.15).

```

Theta1=30
a=Theta1
d1=36
RotZ1=[cosd(a) -sind(a) 0 0;sind(a) cosd(a) 0 0; 0 0 1 0; 0 0 0 1]
TransZd1=[1 0 0 0; 0 1 0 0; 0 0 1 d1; 0 0 0 1]
RotX90=[1 0 0 0; 0 cosd(90) -sind(90) 0; 0 sind(90) cosd(90) 0; 0 0 0 1]
A1= RotZ1*TransZd1*RotX90

Theta2=45
b=Theta2
RotZ2=[cosd(b) -sind(b) 0 0;sind(b) cosd(b) 0 0; 0 0 1 0; 0 0 0 1]
a2=40
TransXa2=[1 0 0 a2; 0 1 0 0; 0 0 1 0; 0 0 0 1]
A2=RotZ2*TransXa2

T2=A1*A2

Theta3=-60
a3=11
RotZ3=[cosd(Theta3) -sind(Theta3) 0 0;sind(Theta3) cosd(Theta3) 0 0; 0 0 1 0; 0 0 0 1]
TransXa3=[1 0 0 a3; 0 1 0 0; 0 0 1 0; 0 0 0 1]
RotY90= [cosd(90) 0 sind(90) 0; 0 1 0 0; -sind(90) 0 cosd(90) 0; 0 0 0 1];
A3= RotZ3*TransXa3*RotY90

T3=A1*A2*A3
Theta4=-20
d4=14
RotZ4=[cosd(Theta4) -sind(Theta4) 0 0;sind(Theta4) cosd(Theta4) 0 0; 0 0 1 0; 0 0 0 1]
TransZd4=[1 0 0 0; 0 1 0 0; 0 0 1 d4; 0 0 0 1]
RotYN90= [cosd(-90) 0 sind(-90) 0; 0 1 0 0; -sind(-90) 0 cosd(-90) 0; 0 0 0 1]
A4= RotZ4*TransZd4*RotYN90

T4=A1*A2*A3*A4
Theta5=-60
RotZ5=[cosd(Theta5) -sind(Theta5) 0 0;sind(Theta5) cosd(Theta5) 0 0; 0 0 1 0; 0 0 0 1]

A5= RotZ5*RotY90

T5= A1*A2*A3*A4*A5

d6=4
Theta6=0
TransZd6=[1 0 0 0; 0 1 0 0; 0 0 1 d6; 0 0 0 1]
A6=TransZd6

T6= A1*A2*A3*A4*A5*A6

```

T6 =

-0.5465	0.7443	0.3839	46.9436
0.7695	0.6272	-0.1203	25.7348
-0.3304	0.2297	-0.9155	54.1519
0	0	0	1.0000

Figure 11: Math Script for the Location of the Verifying Point Based on the Robot World Frame

Figure 12 shows the final location of the Verifying Point after moving the robot with same joint variables used in the math script. By checking the location of the VP, it has the same answer as that of the math script, as shown in Figure 11.

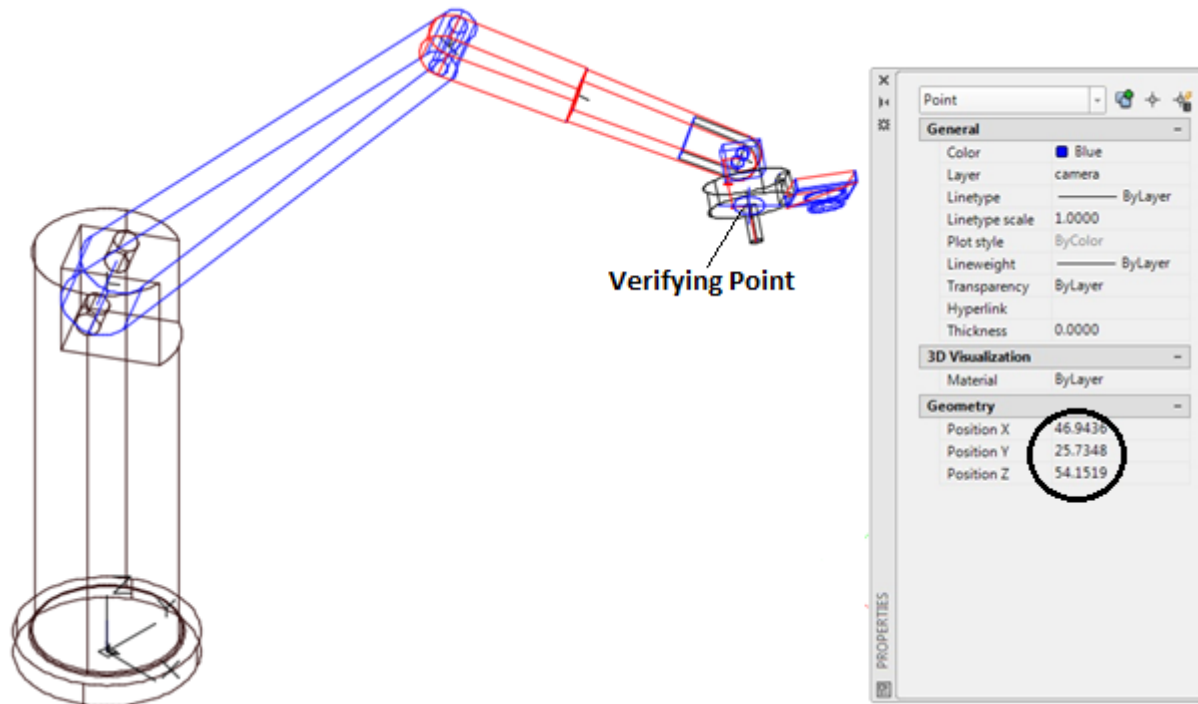


Figure 12: Rotating Robot Arms Based on the Joint Variables and Check the Location of the Verifying Point

### 3.2. Kinematic Transformation for a Mounted Camera

To help student study the kinematic transformation for a machine vision and a robot, Figure 13 shows a camera mounted at the end of Joint 6 with location dimensions.

Equation (19) shows the transformation from  $VP$  to the camera origin:

$$A_{Camera} = Trans(y, 6.5)Trans(x - 0.5)Rot(x, 180^\circ) \quad (19)$$

Equation (20) is applied to calculate the location of the origin of the camera coordinate system based on the world frame:

$$T_{CameraOrigin} = T_{VP} A_{Camera} Trans(x, 0.5) \quad (20)$$

Where  $T_{CameraOrigin}$  is the location of the camera origin based on the robot world frame.

For verifying purpose, Equation (21) shows the transformation from a point (0.5, 0.5) measured from the camera coordinate system to the robot world frame:

$$T_{\text{Camera point}} = T_{VP} A_{\text{Camera}} \text{Trans}(x, 0.5) \text{Trans}(y, 0.5) \quad (21)$$

Result of the transformation in math script is shown in Figure 14 and is verified by the robot, as shown in Figure 15.

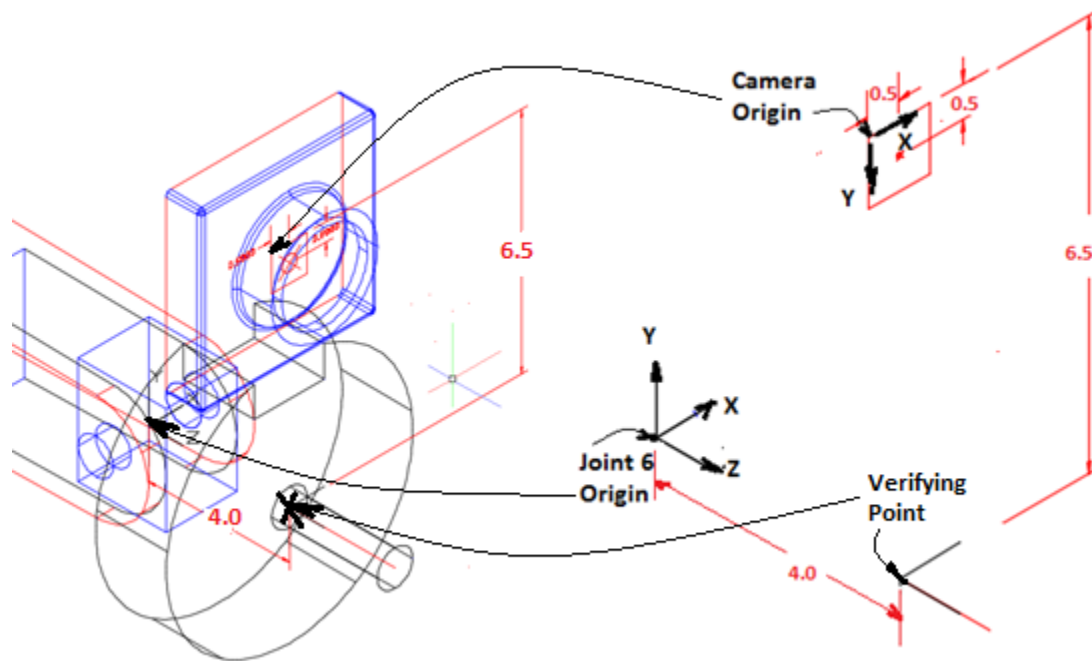


Figure 13: Location of the Camera Origin Based on Joint 6

```

A6=RotZ6*TransZd6

T6= A1*A2*A3*A4*A5*A6

Yc=6.5
TransYc=[1 0 0 0; 0 1 0 Yc; 0 0 1 0; 0 0 0 1]
Xc=-0.5
TransXc=[1 0 0 Xc; 0 1 0 0; 0 0 1 0; 0 0 0 1]
RotX180=[1 0 0 0; 0 cosd(180) -sind(180) 0; 0 sind(180) cosd(180) 0; 0 0 0 1]

Acamera=TransYc*TransXc*RotX180

Tcamera=T6*Acamera

Xp=0.5
TransXp=[1 0 0 Xp; 0 1 0 0; 0 0 1 0; 0 0 0 1]
Yp=0.5
TransYp=[1 0 0 0; 0 1 0 Yp; 0 0 1 0; 0 0 0 1]
Apoint=TransXp*TransYp

Tcamerapoint=Tcamera*Apoint

Tcamerapoint =
    -0.1012    -0.9178    -0.3839    52.4504
         0.9800    -0.1584     0.1203    26.6850
    -0.1713    -0.3641     0.9155    56.3365
         0         0         0         1

```

Figure 14: Math Script for Equation (21)

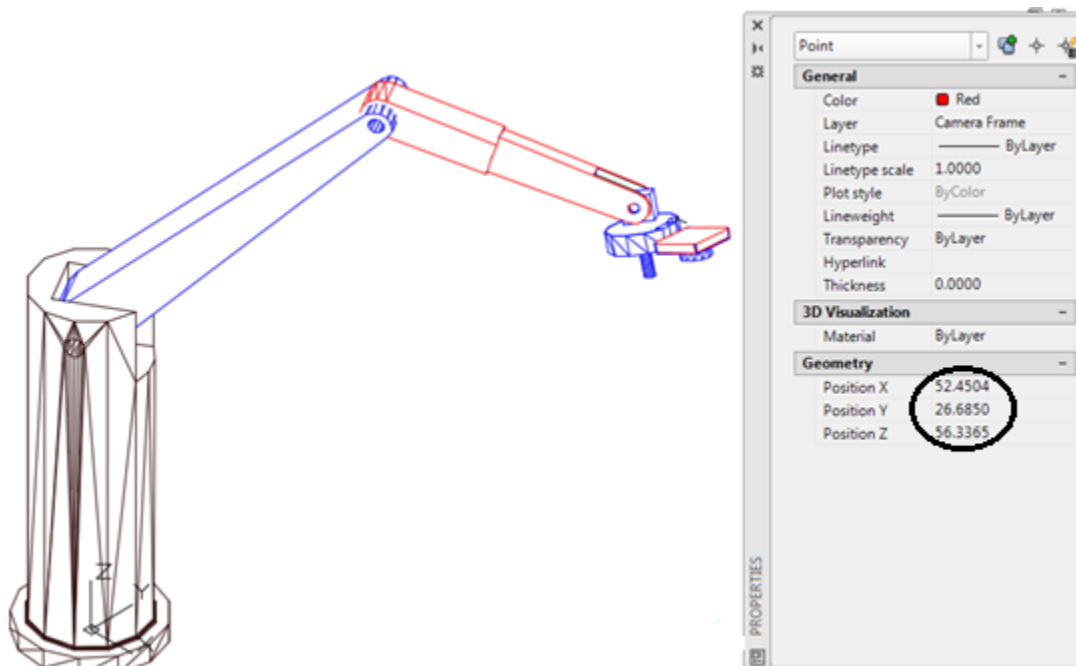


Figure 15: Verifying the Result of Equation (20)

### 3.3. Kinematic Transformation for the Location of an Object from the Mounted Camera to the Robot World Frame



Figure 16 shows that the visualized robot is configured so that it will pick up an object in the vertical (Z axis) direction of the robot world frame. The following joint variables are selected:  $\theta_1 = 0^\circ$ ;  $\theta_2 = 45^\circ$ ;  $\theta_3 = -60^\circ$ ;  $\theta_4 = 0^\circ$ ;  $\theta_5 = -75^\circ$ ;  $\theta_6 = 90^\circ$ . By substituting these joint variables into Equations (12) to (17) and then using Equations (18) to (20), the location of the camera origin,  $T_{CameraOrigin}$ , can be found as (51.9324,-6.500, 53.8138), as shown in Figure 16 from math script.

$$T_{cameraOrigin} = \begin{bmatrix} 1.0000 & 0 & 0.0000 & 51.9324 \\ 0 & 1.0000 & 0 & -6.5000 \\ -0.0000 & 0 & 1.0000 & 53.8138 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

Figure 16: Location of the Camera Origin

An object (sphere) is created with its center located at (30, 10, 2) based on the robot world frame, as shown in Figure 17.

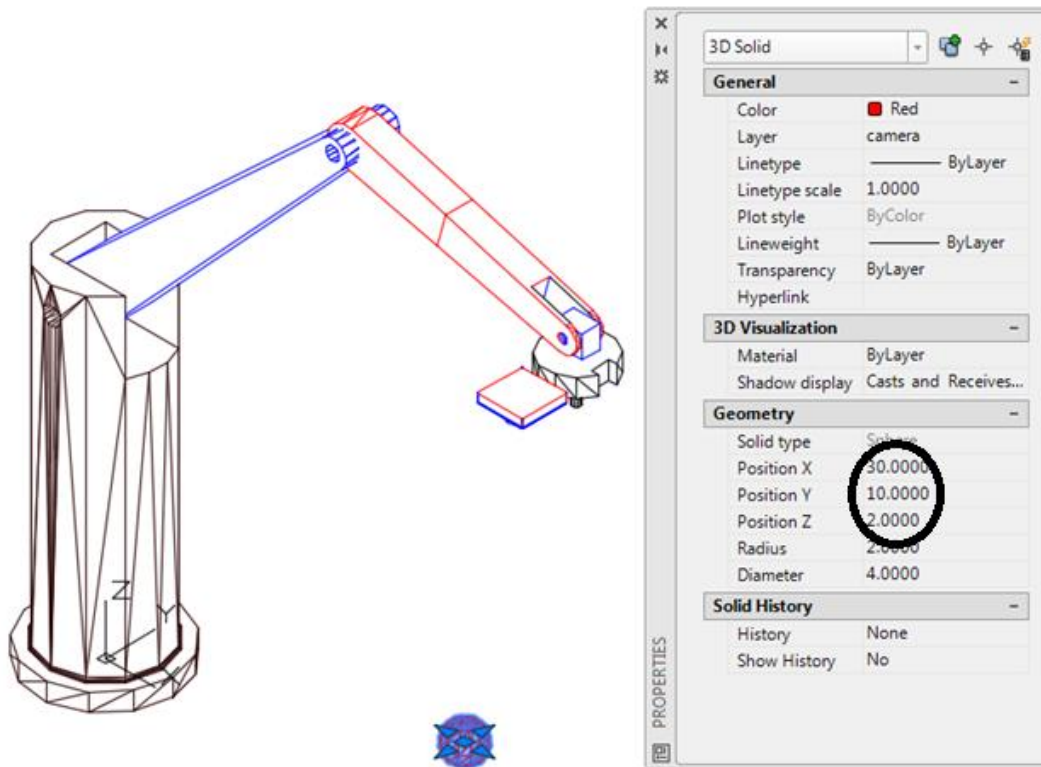


Figure 17: A Sphere Created with Its Center Located at (30, 10, 2)

Figure 18 shows that the center location of the sphere is (-21.9324, 16.5000, -51.8138) measured from camera origin. Since this is the information which can be obtained from a machine vision system, Equation (22) is applied to convert this location to be based on the robot world frame:

$$T_{obj} = T_{CameOrigin} Trans(x, -21.9324) Trans(y, 16.5) Trans(z, -51.8138) \quad (22)$$

Where  $T_{obj}$  represents the location of the sphere based on the robot world frame. Figure 18 shows the result of  $T_{obj}$  calculated from math script. Result shows that using the robot kinematic transformation can calculate the correct location of the sphere verified from Figure 17.

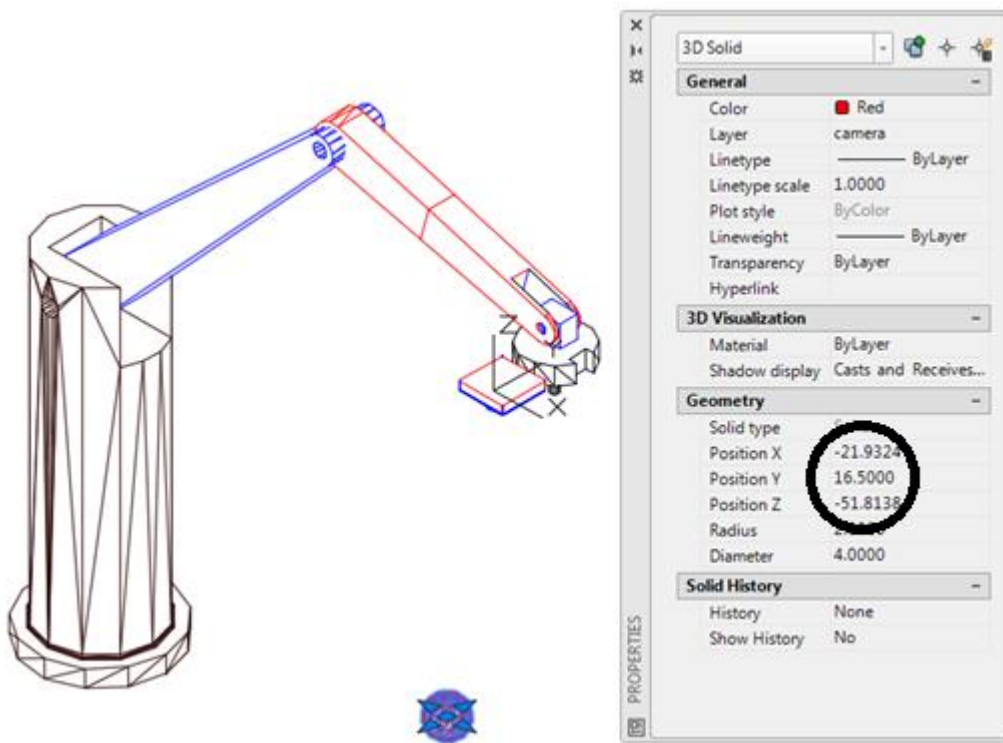


Figure 18: Location of the Sphere When Measured from Camera Origin

$$T_{obj} = \begin{bmatrix} 1.0000 & 0 & 0.0000 & 30.0000 \\ 0 & 1.0000 & 0 & 10.0000 \\ -0.0000 & 0 & 1.0000 & 2.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

Figure 19: Location of the Sphere Calculated from Math Script

By changing the location of the sphere in Figure 17 and verifying the new location from Figure 18, students can repeat the computation using Equation (22).

#### 4. Testing Results

Before the visualized robot was applied in teaching robotic kinematics, most students were not enthusiastic about this topic mainly because of matrices were heavily involved in calculating final answers. As a result, less than 50% of the students passed this topic. After math software and the visualized robot were introduced in the class, students were able to use math software to formulate their kinematic equations and verify their answers using the robot. The test performance was significantly improved with approximately 80% passing rate in this topic. The sample size involved in the test performance assessment is forty eight students.

#### 5. Summary

Combination of the visualized robot and math script provide an effective tool for engineering technology students to study robot kinematic transformation. The math script can be applied to formulate the homogeneous transformation matrices by allowing changing the variables in a matrix. The visualized robot can be utilized to verify the answers for questions created by the instructor. More verifying points can be created at each joint of the robot so that students can have a step-by-step checkup of their kinematic problems. When a camera is attached on the robot and the location of an object is sensed by the camera, the robot and math script can also be applied to verify the coordinate transformations between a robot and a camera.

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